Elastic Scattering of Electrons and Positrons by Screened Nuclei *

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Results of calculations of cross sections for elastic electron and positron scattering are given in angular steps of 15 degrees for elements $Z=6$, 13, 29, 50, 82, and 92 and energies $T=0.2$, 0.4, 0.7, 1, 2, 4, and 10 MeV. The calculation is based on the separability of the cross section into two factors, one describing screening and the other, spin and relativistic effects. The first factor is obtained by the MOLIÈRE approximation [8]. The second factor is taken from a paper by DOGGETT and SPENCER [5]. Different screening potentials for $Z=29$ were applied.

I. Introduction

The cross section for elastic scattering of electrons by a screened nucleus has been obtained by MOTT [1]. The result is given in the form of a LEGENDRE expansion with coefficients determined by the phase shifts the electrons undergo in the scattering field. In a recent survey on electron scattering without atomic or nuclear excitation, MOTZ et al. [2] introduced a rational nomenclature in which MOTT's result is called the MOTT exact phase shift formula (Formula 1A – 109(b) in their paper). In the present report we have used their nomenclature and units; i. e., we measure energies in $m_0 c^2$, momenta in $m_0 c$, and lengths in units of the electron COMPTON wavelength. The formulae are valid for a point charge, infinitely heavy nucleus whose spin effects are negligible.

LIN [3] evaluated numerically the MOTT exact phase shift formula for copper and gold and various energies. He also includes references to previous calculations. MOTZ et al. [4] established good agreement between experiment and theory. It is the screening effect that makes computation so complicated and tedious. For one angle, up to 50 phase shifts have to be obtained by numerical integration. (Thanks to a transformation, the 200 or more phase shifts usually required can be reduced to 50.)

The evaluation of the MOTT exact phase shift formula for an unscreened nucleus (Formula 1A – 109(a) in reference [2]) is simpler, but it must be performed for each element and each electron energy. Tables published by DOGGETT and SPENCER [5] give this cross section in units of the RUTHERFORD cross section. SHERMAN [6] also includes this ratio in his paper on COULOMB scattering of relativistic electrons. Deviation of this ratio,

$$S = \left[\frac{d\sigma}{d\Omega}\right]_{\text{sp, nosc}} \Big/ \left[\frac{d\sigma}{d\Omega}\right]_{\text{nosp, nosc}},$$

from unity must be attributed to effects of the spin on elastic scattering.

The next step is to investigate how screening influences this ratio. Therefore we introduce the ratio

$$S_{\text{ex}} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{sp, sc}} \Big/ \left[\frac{d\sigma}{d\Omega}\right]_{\text{nosp, sc}}.$$

If the deviations of S_{ex} from unity parallel those of S, one could substitute the latter, which is more readily obtainable from DOGGETT and SPENCER's calculations, for S_{ex}, which requires the tedious computations of LIN. Besides this advantage, an agreement of both ratios would bear great physical significance. It would indicate that the screening effects can be factored out so that they cancel each other in the particular ratio. As a consequence, one could

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[1] N. F. MOTT and H. S. W. MASSEY, The Theory of Atomic Collisions, Oxford University Press, London 1949, Second Edition.

[2] J. W. MOTZ, H. OLSEN, and H. W. KOCH, Rev. Mod. Phys. **36**, 881 [1964].

[3] S. R. LIN, Phys. Rev. **133**, A 965 [1964].

[4] J. W. MOTZ, R. C. PLACIOUS, and C. E. DICK, Phys. Rev. **132**, 2558 [1963].

[5] J. A. DOGGETT and L. V. SPENCER, Phys. Rev. **103**, 1597 [1956].

[6] N. SHERMAN, Phys. Rev. **103**, 1601 [1956].

form ratios

$$R = \left[\frac{d\sigma}{d\Omega}\right]_{\text{nosp, sc}} \Big/ \left[\frac{d\sigma}{d\Omega}\right]_{\text{nosp, nosc}}$$

and

$$R_{ex} = \left[\frac{d\sigma}{d\Omega}\right]_{\text{sp, sc}} \Big/ \left[\frac{d\sigma}{d\Omega}\right]_{\text{sp, nosc}}$$

which bring about the influence of screening, while the spin dependence cancels out. The ratio R is relatively simple, since the KLEIN–GORDON equation for a spinless particle suffices, and its denominator is nothting but the RUTHERFORD cross section.

Remaining differences in the two respective ratios must be ascribed to the combined effects of spin and screening on the cross section. This overlapping of both effects or, better, the justification for its neglect, was one of the objectives of a recent paper by ZEITLER and OLSEN [7] showing that for electron energies above 200 keV

$$R_{ex} = R \qquad (1)$$

holds for all elements and all scattering angles with sufficient accuracy. The errors are estimated to be $\xi = 2 \cdot Z^{1/3}/(137)^2 \beta p$. [Eq. (1) implies likewise that $S_{ex} = S$.] The other objective of that paper was to show that the ratio R can be calculated by the so-called MOLIÈRE approximation [8] rather than by the cumbersome phase-shift calculation for a spinless particle [1, 3]. The ultimate justification for extending the MOLIÈRE approximation to all angles is the close agreement between the results of the approximated value of R and the exact values R_{ex} available from LIN's calculations.

Fig. 1 exemplifies the general behavior of the various ratios. In this figure R_{ex} and R for gold and an electron energy of 400 keV are plotted versus the scattering angle. The broken line R_b pertains to the influence of screening as determined by first-order BORN approximation. The ratio S, describing the influence of spin, is also included.

Close agreement between R_{ex} and R, especially at small angles, is obtained. The reason for this is twofold. (1) Spin effects are in any case small at small angles, as can be seen from the fact that curve S in Fig. 1 remains close to unity. Furthermore, overlapping with screening effects does not occur. (2) The MOLIÈRE approximation is a small-angle approximation and therefore most suitable for describing the screening effects at small angles, where they are predominant.

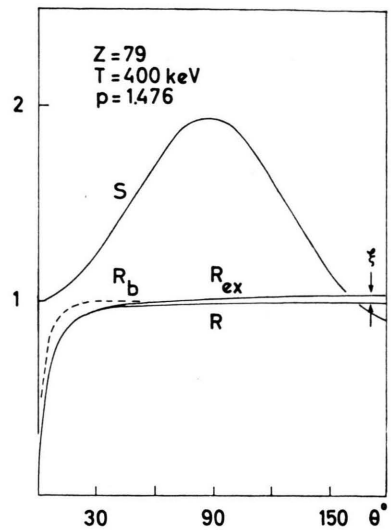

Fig. 1. Ratios of different cross sections as a function of the scattering angle. For notation see text.

For the example given in Fig. 1 a first-order BORN approximation cannot be expected to give reasonable values because the crucial parameter

$$a = Z/137\,\beta = 0.696$$

is not small compared to unity and therefore does not meet the requirement for the application of this approximation. The resulting errors of R_b at small angles are large. At very large angles both approximations, MOLIÈRE and BORN, give the same limit for R, namely, unity, and hence the same deviations from the exact value R_{ex}.

In summary, the MOLIÈRE approximation avoids the large errors of the BORN approximation at small angles while maintaining a high degree of accuracy at large angles.

The purpose of this paper is to present actual calculations of cross sections for electrons scattered elastically by screened nuclei, based on the possibility [7] of writing the exact cross section as a product

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{sp, sc}} = R\left[\frac{d\sigma}{d\Omega}\right]_{\text{sp, nosc}} \qquad (2)$$

in which case the factor R depends on screening only and is produced by MOLIÈRE's approximation. The other factor depends on spin and relativistic effects only and is taken from DOGGETT and SPENCER's article [5].

[7] E. ZEITLER and H. OLSEN, Phys. Rev. **136**, A 1546 [1964].

[8] G. MOLIÈRE, Z. Naturforschg. **2 a**, 133 [1947].

Furthermore, in this paper we shall give cross sections for positrons, making use of the fact that Eqs. (1) and (2) hold not only for electrons but also for positrons. Since the "screening factor" is independent of the charge of the scattered particle, the calculation of the exact cross sections for elastic positron scattering require additionally only the "spin factors," that is, the cross sections for elastic positron scattering by an *unscreened* nucleus. The latter, however, are also available from DOGGETT and SPENCER's results [5].

II. Method of Calculation

The present method of calculation provides values of cross sections that are sufficiently accurate for most experimental applications for all elements and all angles in a wide range of energies.

Table 1 gives the errors ξ inherent in the method of calculation as outlined in Section I for the elements and energies considered in this paper.

For elements $Z<29$, self-consistent potentials approximated by STRAND and BONHAM [9] were used; for elements $Z>29$, THOMAS–FERMI potentials approximated according to MOLIÈRE [8] were used. For $Z=29$ the self-consistent potential given by BYATT [10] and applied by LIN to his calculations for electrons was also used. All potentials are written in the following form:

$$V(r) = -\frac{Z}{137\,r}\sum_{1}^{3}\left(a_i\,e^{-b_i\Lambda r}+c_i\,\Lambda\,r\,e^{-d_i\Lambda r}\right). \quad (3)$$

In the case of the self-consistent potentials, Λ equals $1/137$, and, in the case of the THOMAS–FERMI potential, Λ equals $Z^{1/3}/0.885\cdot 137$. In applying two values for Λ, the parameters a_i, b_i, c_i, and d_i can be taken directly from the respective publication [8–10]; they are compiled in Table 2. When $y=q/\Lambda$ is introduced as the angular variable, whereby q is the momentum transfer $2\,p\sin\Theta/2$, the ratio R is represented by two integrals, Q_1 and Q_2.

$$R = Q_1^2+Q_2^2, \quad (4)$$

$$Q_1 = y\int_0^\infty x\,dx\,J_1(x\,y)\,f(x)\cos(2\,a\,\varphi(x)), \quad (5\,a)$$

$$Q_2 = -y\int_0^\infty x\,dx\,J_1(x\,y)\,f(x)\sin(2\,a\,\varphi(x)), \quad (5\,b)$$

$$f(x) = \sum_{1}^{3}\left[a_i\,b_i\,K_1(b_i\,x)+c_i\,d_i\,x\,K_0(d_i\,x)\right],$$

$$\varphi(x) = \sum_{1}^{3}\left[a_i\,K_0(b_i\,x)\;+c_i\,x\,K_1(d_i\,x)\right]$$

Here J_k and K_k are the BESSEL function and the modified HANKEL function of the k-th order.

$1-\beta$	0.0012	0.0064	0.0209	0.0589	0.0934	0.1721	0.3047
p	20.546	8.771	4.811	2.783	2.149	1.497	0.9675
Z \ T(MeV)	10	4	2	1	0.7	0.4	0.2
6	0.006	0.013	0.025	0.044	0.060	0.094	0.173
13	0.016	0.037	0.069	0.124	0.167	0.263	0.484
29	0.043	0.109	0.202	0.363	0.487	0.766	1.411
50	0.096	0.225	0.417	0.749	1.008	1.584	2.918
82	0.185	0.436	0.806	1.450	1.949	3.064	5.643
92	0.256	0.509	0.940	1.690	2.272	3.572	6.579

Table 1. Values for $100\,\xi = 100\,Z^{4/3}/(137)^2\,(2/\beta\,p)$.

Z	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3	$137\,\Lambda$
6*	1.3391	−0.3391		1.7315	13.713		−2.7379	−2.0444		4.718	8.333		
13*	1.1951	−0.1951		1.3532	27.522		−2.3888	−4.804		2.674	14.036		1
29	1.5436	−0.5436		3.856	47.41		−8.439	−16.284	0.1973	9.935	32.013	1.4838	1
29+	0.22	0.78		0.319	1.081								2.719
50	0.35	0.55	0.1	0.3	1.2	6.0							3.260
82	0.35	0.55	0.1	0.3	1.2	6.0							3.845
92	0.35	0.55	0.1	0.3	1.2	6.0							3.995

* STRAND and BONHAM [9], + BYATT [10], all others, THOMAS–FERMI according to MOLIÈRE [8]

Table 2. Parameters in screened potentials; Eq. (1).

[9] T. G. STRAND and R. A. BONHAM, J. Chem. Phys. **40**, 1686 [1964].

[10] W. J. BYATT, Phys. Rev. **104**, 1298 [1956].

In most of the cases the angular variable y is so large that an approximate method [7] for calculating R is applicable. This method leads to arithmetic formulae for Q_1 and Q_2.

$$Q_1 = 1 - [\Phi_1 + \Phi_2 \ln(y/2) - P)] \cdot 4\, a^2/y^2 + \Phi_2(3\, a^2 - 1)/y^2, \tag{6a}$$

$$Q_2 = -[\Phi_1 + \Phi_2\, (\ln(y/2) - P)] \cdot 2\, a(1 - a^2)/y^2 + \Phi_2 \cdot 4\, a/y^2\,. \tag{6b}$$

T(MeV)	10	4	2	1	0.7	0.4	0.2
Θ(deg.)				$Z = 6$			
15.0	0.585 E-07*	0.324 E-06	0.111 E-05	0.359 E-05	0.650 E-05	0.165 E-04	0.531 E-04
30.0	0.364 E-08	0.202 E-07	0.692 E-07	0.225 E-06	0.408 E-06	0.104 E-05	0.348 E-05
45.0	0.704 E-09	0.391 E-08	0.134 E-07	0.439 E-07	0.801 E-07	0.208 E-06	0.703 E-06
60.0	0.214 E-09	0.119 E-08	0.411 E-08	0.136 E-07	0.250 E-07	0.659 E-07	0.229 E-06
75.0	0.824 E-10	0.459 E-09	0.160 E-08	0.536 E-08	0.100 E-07	0.271 E-07	0.974 E-07
90.0	0.362 E-10	0.202 E-09	0.711 E-09	0.245 E-08	0.465 E-08	0.131 E-07	0.494 E-07
105.0	0.170 E-10	0.958 E-10	0.343 E-09	0.123 E-08	0.240 E-08	0.714 E-08	0.285 E-07
120.0	0.813 E-11	0.464 E-10	0.171 E-09	0.653 E-09	0.134 E-08	0.427 E-08	0.183 E-07
135.0	0.371 E-11	0.217 E-10	0.851 E-10	0.364 E-09	0.797 E-09	0.280 E-08	0.129 E-07
150.0	0.145 E-11	0.908 E-11	0.410 E-10	0.214 E-09	0.516 E-09	0.202 E-08	0.101 E-07
165.0	0.362 E-12	0.302 E-11	0.197 E-10	0.141 E-09	0.378 E-09	0.163 E-08	0.862 E-08
180.0	0.408 E-13	0.122 E-11	0.134 E-10	0.120 E-09	0.337 E-09	0.151 E-08	0.819 E-08
				$Z = 13$			
15.0	0.280 E-06	0.155 E-05	0.532 E-05	0.172 E-04	0.311 E-04	0.790 E-04	0.244 E-03
30.0	0.177 E-07	0.984 E-07	0.337 E-06	0.109 E-05	0.198 E-05	0.505 E-05	0.168 E-04
45.0	0.348 E-08	0.193 E-07	0.663 E-07	0.216 E-06	0.394 E-06	0.102 E-05	0.342 E-05
60.0	0.107 E-08	0.594 E-08	0.205 E-07	0.675 E-07	0.124 E-06	0.325 E-06	0.112 E-05
75.0	0.415 E-09	0.231 E-08	0.801 E-08	0.268 E-07	0.498 E-07	0.134 E-06	0.477 E-06
90.0	0.183 E-09	0.103 E-08	0.360 E-08	0.123 E-07	0.233 E-07	0.650 E-07	0.242 E-06
105.0	0.868 E-10	0.489 E-09	0.174 E-08	0.617 E-08	0.120 E-07	0.353 E-07	0.139 E-06
120.4	0.417 E-10	0.237 E-09	0.869 E-09	0.328 E-08	0.667 E-08	0.210 E-07	0.889 E-07
135.0	0.190 E-10	0.111 E-09	0.430 E-09	0.182 E-08	0.394 E-08	0.136 E-07	0.625 E-07
150.0	0.745 E-11	0.463 E-10	0.205 E-09	0.105 E-08	0.251 E-08	0.972 E-08	0.483 E-07
165.0	0.187 E-11	0.151 E-10	0.962 E-10	0.680 E-09	0.181 E-08	0.778 E-08	0.411 E-07
180.0	0.191 E-12	0.579 E-11	0.634 E-10	0.570 E-09	0.160 E-08	0.720 E-08	0.390 E-07
				$Z = 29$			
15.0	0.146 E-05	0.809 E-05	0.277 E-04	0.892 E-04	0.160 E-03	0.403 E-03	0.129 E-02
30.0	0.968 E-07	0.536 E-06	0.183 E-05	0.592 E-05	0.107 E-04	0.271 E-04	0.882 E-04
45.0	0.197 E-07	0.109 E-06	0.375 E-06	0.122 E-05	0.220 E-05	0.563 E-05	0.186 E-04
60.0	0.627 E-08	0.348 E-07	0.120 E-06	0.391 E-06	0.714 E-06	0.185 E-05	0.623 E-05
75.0	0.250 E-08	0.139 E-07	0.480 E-07	0.159 E-06	0.293 E-06	0.775 E-06	0.269 E-05
90.0	0.113 E-08	0.630 E-08	0.220 E-07	0.742 E-07	0.139 E-06	0.379 E-06	0.137 E-05
105.0	0.544 E-09	0.305 E-08	0.108 E-07	0.375 E-07	0.720 E-07	0.206 E-06	0.790 E-06
120.0	0.265 E-09	0.150 E-08	0.542 E-08	0.199 E-07	0.398 E-07	0.122 E-06	0.500 E-06
135.0	0.122 E-09	0.707 E-09	0.268 E-08	0.109 E-07	0.231 E-07	0.776 E-07	0.348 E-06
150.0	0.480 E-10	0.293 E-09	0.125 E-08	0.610 E-08	0.142 E-07	0.540 E-07	0.265 E-06
165.0	0.119 E-10	0.916 E-10	0.554 E-09	0.378 E-08	0.994 E-08	0.424 E-07	0.224 E-06
180.0	0.103 E-11	0.310 E-10	0.344 E-09	0.307 E-08	0.865 E-08	0.389 E-07	0.211 E-06
				$Z = 50$			
15.0	0.457 E-05	0.252 E-04	0.855 E-04	0.270 E-03	0.475 E-03	0.115 E-02	0.340 E-02
30.0	0.325 E-06	0.180 E-05	0.612 E-05	0.196 E-04	0.349 E-04	0.858 E-04	0.264 E-03
45.0	0.711 E-07	0.394 E-06	0.134 E-05	0.432 E-05	0.775 E-05	0.193 E-04	0.604 E-04
60.0	0.241 E-07	0.133 E-06	0.456 E-06	0.148 E-05	0.266 E-05	0.672 E-05	0.215 E-04
75.0	0.101 E-07	0.562 E-07	0.193 E-06	0.630 E-06	0.115 E-05	0.295 E-05	0.977 E-05
90.0	0.478 E-08	0.266 E-07	0.920 E-07	0.305 E-06	0.563 E-06	0.149 E-05	0.514 E-05
105.0	0.239 E-08	0.133 E-07	0.466 E-07	0.159 E-06	0.299 E-06	0.826 E-06	0.301 E-05
120.0	0.119 E-08	0.672 E-08	0.240 E-07	0.854 E-07	0.167 E-06	0.490 E-06	0.193 E-05
135.0	0.564 E-09	0.322 E-08	0.120 E-07	0.464 E-07	0.960 E-07	0.310 E-06	0.134 E-05
150.0	0.224 E-09	0.134 E-08	0.550 E-08	0.254 E-07	0.578 E-07	0.212 E-06	0.102 E-05
165.0	0.552 E-10	0.402 E-09	0.228 E-08	0.148 E-07	0.386 E-07	0.163 E-06	0.856 E-06
180.0	0.389 E-11	0.118 E-09	0.130 E-08	0.116 E-07	0.328 E-07	0.148 E-06	0 807 E-06

Tab. 3.

T(MeV)	10	4	2	1	0.7	0.4	0.2
Θ(deg.)				$Z = 82$			
15.0	0.123 E-04	0.675 E-04	0.226 E-03	0.692 E-03	0.120 E-02	0.278 E-03	0.768 E-01
30.0	0.960 E-06	0.529 E-05	0.179 E-04	0.557 E-04	0.974 E-04	0.226 E-03	0.640 E-03
45.0	0.245 E-06	0.135 E-05	0.458 E-05	0.143 E-04	0.252 E-04	0.592 E-04	0.165 E-03
60.0	0.974 E-07	0.538 E-06	0.182 E-05	0.574 E-05	0.101 E-04	0.240 E-04	0.687 E-04
75.0	0.472 E-07	0.261 E-06	0.888 E-06	0.282 E-05	0.501 E-05	0.119 E-04	0.356 E-04
90.0	0.251 E-07	0.139 E-06	0.476 E-06	0.153 E-05	0.275 E-05	0.682 E-05	0.209 E-04
105.0	0.138 E-07	0.769 E-07	0.265 E-06	0.871 E-06	0.159 E-05	0.411 E-05	0.134 E-04
120.0	0.747 E-08	0.417 E-07	0.146 E-06	0.499 E-06	0.941 E-06	0.258 E-05	0.918 E-05
135.0	0.372 E-08	0.211 E-07	0.763 E-07	0.280 E-06	0.558 E-06	0.168 E-05	0.671 E-05
150.0	0.154 E-08	0.901 E-08	0.355 E-07	0.153 E-06	0.334 E-06	0.116 E-05	0.527 E-05
165.0	0.380 E-09	0.263 E-08	0.138 E-07	0.849 E-07	0.216 E-06	0.890 E-06	0.452 E-05
180.0	0.216 E-10	0.647 E-09	0.714 E-08	0.638 E-07	0.179 E-06	0.806 E-06	0.429 E-05
				$Z = 92$			
15.0	0.151 E-04	0.830 E-04	0.277 E-03	0.842 E-03	0.145 E-02	0.335 E-02	0.920 E-02
30.0	0.117 E-05	0.647 E-05	0.218 E-04	0.674 E-04	0.117 E-03	0.269 E-03	0.746 E-03
45.0	0.317 E-06	0.175 E-05	0.588 E-05	0.182 E-04	0.316 E-04	0.726 E-04	0.195 E-03
60.0	0.135 E-06	0.747 E-06	0.252 E-05	0.784 E-05	0.137 E-04	0.316 E-04	0.856 E-04
75.0	0.703 E-07	0.388 E-06	0.131 E-05	0.412 E-05	0.723 E-05	0.170 E-04	0.473 E-04
90.0	0.397 E-07	0.219 E-06	0.746 E-06	0.237 E-05	0.420 E-05	0.101 E-04	0.295 E-04
105.0	0.228 E-07	0.127 E-06	0.434 E-06	0.141 E-05	0.254 E-05	0.638 E-05	0.198 E-04
120.0	0.128 E-07	0.712 E-07	0.248 E-06	0.833 E-06	0.155 E-05	0.414 E-05	0.141 E-04
135.0	0.653 E-08	0.369 E-07	0.132 E-06	0.477 E-06	0.938 E-06	0.276 E-05	0.106 E-04
150.0	0.274 E-08	0.160 E-07	0.622 E-07	0.262 E-06	0.567 E-06	0.193 E-05	0.848 E-05
165.0	0.683 E-09	0.465 E-08	0.241 E-07	0.145 E-06	0.366 E-06	0.148 E-05	0.736 E-05
180.0	0.363 E-10	0.110 E-08	0.121 E-07	0.108 E-06	0.302 E-06	0.134 E-05	0.701 E-05

* The decimal exponent E-07 means a factor 10^{-7}.

Table 3. Cross section for elastic electron scattering in units of 10^8 barns.

In Eqs. (6 a) and (6 b)

$$\Phi_1 = \sum_1^3 \left[a_i b_i^2 \left(\ln \frac{2}{\gamma b_i} + 1 \right) - 2 c_i d_i \ln \frac{2}{\gamma d_i} \right],$$

$$\Phi_2 = \sum_1^3 [a_i b_i^2 - 2 c_i d_i]; \qquad P = \Re\mathrm{e}[\psi(i a)]$$

with ψ the logarithmic derivative of the Γ-function and $\gamma = 1.7807$ EULER's constant.

While Q_1 and Q_2 are derived from the real and the imaginary parts of the T-matrix, the first-order BORN approximation gives a ratio R_b, which is the square of a single expression, namely,

$$Q_b = y^2 \sum_1^3 \left[\frac{a_i}{b_i^2 + y^2} + \frac{2 c_i d_i}{(d_i^2 + y^2)^2} \right]. \qquad (7)$$

For large values of y, Eq. (7) can be brought into the form

$$Q_b = 1 - (1/y^2)\, \Phi_2, \qquad (8)$$

which is the coefficient of a^0 when Q_1 in Eq. (6 a) is regarded as a power series in a.

The computations were programmed in Fortran and run on an IBM 7090 computer.

III. Results and Discussion

Table 3 presents the cross sections for elastic electron scattering in units of 10^8 barns. Since we did not repeat the DOGGETT–SPENCER calculation, we had to use their steps in scattering angles of 15 degrees. The over-all accuracy is determined by the errors due to the replacement of R_{ex} by R, which are already introduced in Table 1, and the errors inherent in DOGGETT and SPENCER's calculation. They estimate that except for angles close to 180 degrees at high energy, the error should be less than 0.5 per cent.

Table 4 presents the cross sections for elastic positron scattering in units of 10^8 barns.

To investigate the sensitivity of the cross section to the choice of the potential, we applied two different approximations of HARTREE potentials, one according to BYATT and the other according to STRAND and BONHAM. BYATT claims an error of only 5 per cent out to a distance at which the potential expressed in units of the COULOMB potential has fallen to about 1 per cent. STRAND and BONHAM, on the other hand, give only the standard deviation of 0.2

for their least-square fit of radial electron densities. For large distances from the nucleus the accuracy of their potentials is lower than that of BYATT, whereas at small distances it is clearly superior. For small-angle calculations more accurate potentials are desirable.

T(MeV)	10	4	2	1	0.7	0.4	0.2
Θ(deg.)				$Z = 6$			
15.0	0.551 E-07*	0.306 E-06	0.105 E-05	0.340 E-05	0.616 E-05	0.158 E-04	0.509 E-04
30.0	0.339 E-08	0.188 E-07	0.646 E-07	0.210 E-06	0.383 E-06	0.986 -E06	0.332 E-05
45.0	0.648 E-09	0.360 E-08	0.124 E-07	0.407 E-07	0.745 E-07	0.194 E-06	0.666 E-06
60.0	0.194 E-09	0.108 E-08	0.374 E-08	0.124 E-07	0.230 E-07	0.613 E-07	0.216 E-06
75.0	0.737 E-10	0.412 E-09	0.143 E-08	0.486 E-08	0.913 E-08	0.251 E-07	0.919 E-07
90.0	0.321 E-10	0.180 E-09	0.636 E-09	0.221 E-08	0.425 E-08	0.122 E0-7	0.467 E-07
105.0	0.150 E-10	0.849 E-10	0.305 E-09	0.111 E-08	0.220 E-08	0.618 E-08	0.272 E-07
120.0	0.716 E-11	0.410 E-10	0.152 E-09	0.597 E-09	0.124 E-08	0.404 E-08	0.177 E-07
135.0	0.324 E-11	0.191 E-10	0.763 E-10	0.337 E-09	0.751 E-09	0.268 E-08	0.126 E-07
150.0	0.126 E-11	0.804 E-11	0.375 E-10	0.203 E-09	0.496 E-09	0.197 E-08	0.994 E-08
165.0	0.331 E-12	0.284 E-11	0.191 E-10	0.140 E-09	0.375 E-09	0.162 E-08	0.861 E-08
180.0	0.408 E-13	0.122 E-11	0.134 E-10	0.120 E-09	0.337 E-09	0.151 E-08	0.819 E-08
				$Z = 13$			
15.0	0.258 E-06	0.143 E-05	0.491 E-05	0.159 E-04	0.289 E-04	0.738 E-04	0.231 E-03
30.0	0.156 E-07	0.865 E-07	0.297 E-06	0.967 E-06	0.177 E-05	0.456 E-05	0.154 E-04
45.0	0.294 E-08	0.163 E-07	0.563 E-07	0.184 E-06	0.339 E-06	0.889 E-06	0.307 E-05
60.0	0.874 E-09	0.487 E-08	0.169 E-07	0.560 E-07	0.104 E-06	0.279 E-06	0.993 E-06
75.0	0.330 E-09	0.184 E-08	0.644 E-08	0.218 E-07	0.412 E-07	0.114 E-06	0.422 E-06
90.0	0.143 E-09	0.803 E-09	0.284 E-08	0.991 E-08	0.192 E-07	0.554 E-07	0.215 E-06
105.0	0.666 E-10	0.377 E-09	0.136 E-08	0.497 E-08	0.997 E-08	0.305 E-07	0.126 E-06
120.0	0.316 E-10	0.181 E-09	0.681 E-09	0.268 E-08	0.565 E-08	0.186 E-07	0.821 E-07
135.0	0.143 E-10	0.849 E-10	0.343 E-09	0.153 E-08	0.345 E-08	0.125 E-07	0.593 E-07
150.0	0.558 E-11	0.350 E-10	0.170 E-09	0.931 E-09	0.231 E-08	0.923 E-08	0.468 E-07
165.0	0.144 E-11	0.128 E-10	0.879 E-10	0.644 E-09	0.176 E-08	0.764 E-08	0.407 E-07
180.0	0.191 E-12	0.575 E-11	0.631 E-10	0.558 E-09	0.160 E-08	0.718 E-08	0.388 E-07
				$Z = 29$			
15.0	0.126 E-05	0.697 E-05	0.237 E-04	0.775 E-04	0.140 E-03	0.357 E-03	0.117 E-02
30.0	0.739 E-07	0.410 E-06	0.141 E-05	0.461 E-05	0.848 E-05	0.218 E-04	0.740 E-04
45.0	0.137 E-07	0.759 E-07	0.262 E-06	0.867 E-06	0.159 E-05	0.422 E-05	0.147 E-04
60.0	0.408 E-08	0.224 E-07	0.776 E-07	0.261 E-06	0.486 E-06	0.132 E-05	0.477 E-05
75.0	0.150 E-08	0.840 E-08	0.295 E-07	0.101 E-06	0.192 E-06	0.540 E-06	0.203 E-05
90.0	0.644 E-09	0.363 E-08	0.129 E-07	0.460 E-07	0.895 E-07	0.264 E-06	0.104 E-05
105.0	0.299 E-09	0.169 E-08	0.618 E-08	0.231 E-07	0.469 E-07	0.146 E-06	0.617 E-06
120.0	0.141 E-09	0.814 E-09	0.309 E-08	0.126 E-07	0.269 E-07	0.907 E-07	0.408 E-06
135.0	0.639 E-10	0.383 E-09	0.158 E-08	0.736 E-08	0.168 E-07	0.620 E-07	0.298 E-06
150.0	0.249 E-10	0.165 E-09	0.810 E-09	0.465 E-08	0.118 E-07	0.468 E-07	0.239 E-06
165.0	0.641 E-11	0.604 E-10	0.438 E-09	0.333 E-08	0.903 E-08	0.393 E-07	0.209 E-06
180.0	0.992 E-12	0.299 E-10	0.330 E-09	0.294 E-08	0.828 E-08	0.370 E-07	0.200 E-06
				$Z = 50$			
15.0	0.365 E-05	0.202 E-04	0.688 E-04	0.219 E-03	0.391 E-03	0.973 E-03	0.300 E-02
30.0	0.210 E-06	0.117 E-05	0.401 E-05	0.131 E-04	0.239 E-04	0.614 E-04	0.204 E-03
45.0	0.383 E-07	0.213 E-06	0.736 E-06	0.244 E-05	0.451 E-05	0.120 E-04	0.414 E-04
60.0	0.111 E-07	0.620 E-07	0.216 E-06	0.730 E-06	0.137 E-05	0.376 E-05	0.136 E-04
75.0	0.412 E-08	0.231 E-07	0.815 E-07	0.283 E-06	0.543 E-06	0.155 E-05	0.588 E-05
90.0	0.176 E-08	0.992 E-08	0.356 E-07	0.129 E-06	0.254 E-06	0.761 E-06	0.306 E-05
105.0	0.810 E-09	0.462 E-08	0.171 E-07	0.656 E-07	0.134 E-06	0.430 E-06	0.183 E-05
120.0	0.381 E-09	0.222 E-08	0.864 E-08	0.363 E-07	0.788 E-07	0.270 E-06	0.123 E-05
135.0	0.173 E-09	0.105 E-08	0.447 E-08	0.217 E-07	0.504 E-07	0.189 E-06	0.907 E-06
150.0	0.678 E-10	0.461 E-09	0.237 E-08	0.142 E-07	0.357 E-07	0.146 E-06	0.735 E-06
165.0	0.178 E-10	0.179 E-09	0.136 E-08	0.105 E-07	0.287 E-07	0.124 E-06	0.649 E-06
180.0	0.319 E-11	0.962 E-10	0.106 E-08	0.945 E-08	0.265 E-07	0.118 E-06	0.623 E-06

Tab. 4.

T(MeV)	10	4	2	1	0.7	0.4	0.2
Θ(deg.)				$Z = 82$			
15.0	0.969 E-05	0.533 E-04	0.180 E-03	0.562 E-03	0.993 E-03	0.239 E-02	0.697 E-01
30.0	0.551 E-06	0.305 E-05	0.105 E-04	0.341 E-04	0.618 E-04	0.156 E-03	0.505 E-03
45.0	0.990 E-07	0.551 E-06	0.191 E-05	0.634 E-05	0.117 E-04	0.310 E-04	0.105 E-03
60.0	0.285 E-07	0.159 E-06	0.556 E-06	0.190 E-05	0.358 E-05	0.984 E-05	0.345 E-04
75.0	0.105 E-07	0.589 E-07	0.209 E-06	0.735 E-06	0.142 E-05	0.409 E-05	0.155 E-04
90.0	0.444 E-08	0.252 E-07	0.914 E-07	0.337 E-06	0.671 E-06	0.204 E-05	0.816 E-05
105.0	0.204 E-08	0.117 E-07	0.440 E-07	0.174 E-06	0.362 E-06	0.117 E-05	0.495 E-05
120.0	0.960 E-09	0.566 E-08	0.226 E-07	0.984 E-07	0.217 E-06	0.755 E-06	0.337 E-05
135.0	0.434 E-09	0.270 E-08	0.119 E-07	0.605 E-07	0.143 E-06	0.538 E-06	0.252 E-05
150.0	0.171 E-09	0.121 E-08	0.660 E-08	0.412 E-07	0.105 E-06	0.423 E-06	0.206 E-05
165.0	0.473 E-10	0.513 E-09	0.407 E-08	0.319 E-07	0.860 E-07	0.366 E-06	0.183 E-05
180.0	0.102 E-10	0.301 E-09	0.330 E-08	0.291 E-07	0.803 E-07	0.350 E-06	0.177 E-05
				$Z = 92$			
15.0	0.122 E-04	0.670 E-04	0.225 E-03	0.698 E-03	0.123 E-02	0.293 E-02	0.837 E-02
30.0	0.692 E-06	0.383 E-05	0.131 E-04	0.426 E-04	0.770 E-04	0.197 E-03	0.620 E-03
45.0	0.124 E-06	0.692 E-06	0.239 E-05	0.796 E-05	0.147 E-04	0.387 E-04	0.131 E-03
60.0	0.357 E-07	0.199 E-06	0.697 E-06	0.238 E-05	0.449 E-05	0.123 E-04	0.440 E-04
75.0	0.131 E-07	0.736 E-07	0.262 E-06	0.922 E-06	0.178 E-05	0.514 E-05	0.193 E-04
90.0	0.556 E-08	0.315 E-07	0.114 E-06	0.423 E-06	0.847 E-06	0.258 E-05	0.102 E-04
105.0	0.255 E-08	0.147 E-07	0.552 E-07	0.219 E-06	0.459 E-06	0.146 E-05	0.622 E-05
120.0	0.120 E-08	0.710 E-08	0.284 E-07	0.125 E-06	0.276 E-06	0.962 E-06	0.425 E-05
135.0	0.542 E-09	0.339 E-08	0.152 E-07	0.778 E-07	0.184 E-06	0.689 E-06	0.319 E-05
150.0	0.213 E-09	0.153 E-08	0.846 E-08	0.533 E-07	0.136 E-06	0.544 E-06	0.261 E-05
165.0	0.604 E-10	0.665 E-09	0.533 E-08	0.417 E-07	0.112 E-06	0.473 E-06	0.233 E-05
180.0	0.131 E-10	0.398 E-09	0.436 E-08	0.381 E-07	0.105 E-06	0.451 E-06	0.224 E-05

* The decimal exponent E-07 means a factor 10^{-7}.

Table 4. Cross sections for elastic positron scattering in units of 10^8 barns.

Θ	R_B	R_S
15	0.9651	0.9419
30	0.9905	0.9596
45	0.9961	1.007
60	0.9989	1.018

Table 5. Comparison of R-ratios with different potentials for copper at 200 keV. R_B = Byatt potential, R_S = Strand and Bonham potential.

Table 5 shows a comparison of R-values for copper at 200 keV calculated with the two different approximations for the potential. This comparison confirms the finding of Lin, namely, that the choice of the potential has a bearing on the cross section in an angular region up to 30 degrees. Yet this range reflects only the range where the two approximations differ. It is remarkable, however, to what great angles screening in general influences the cross section. Both influences, that of screening and that of choice of screening potential, diminish with increasing electron energy.

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